The Hall effect in the normal state of the t-J model

1993 J. Phys.: Condens. Matter 5 L23

(http://iopscience.iop.org/0953-8984/5/3/001)

J. Phys.: Condens. Matter 5 (1993) L23–L28. Printed in the UK

LETTER TO THE EDITOR

The Hall effect in the normal state of the t–J model

S M Manning and Y Chen

Department of Mathematics, Imperial College, 180 Queen's Gate, London SW7 2BZ, UK

Received 3 August 1992

Abstract. In this article we derive, using the gauge approach to the $t-J$ model, the Hall conductivity (σ_H) and Hall coefficient (R_H) for the physical electrons. We find that for sufficiently low doping (δ), $R_H \propto \delta^{-1}$, while for higher doping (at fixed temperature), R_H changes sign.

The Hall coefficient of a conventional Fermi liquid is independent of temperature, while its sign determines the type of charge carrier involved. However, experiments on the normal state of the high-T_c cuprate superconductors reveal a far richer behaviour, as the Hall coefficient now depends on the temperature and doping of the sample, as well as the system considered [1, 2].

The temperature dependence of R_H in $YBa_2Cu_3O_{7-\delta}$ is fitted well by $1/T +$ constant, as it is for $La_{2-\delta}Sr_\delta CuO_4$ when δ lies in the range $0.14 \leqslant \delta \leqslant 0.18$, the superconducting composition range [1, 3, 4]. However, for $La_{2-\delta}Sr_\delta CuO_4$ when $\delta \leqslant 0.1$, but finite, the Hall coefficient displays almost no temperature dependence [2, 4].

Another feature of the Hall coefficient is its sign, which relates to the sign of the charge carrier involved in the relevant transport processes. For $La_{2-\delta}Sr_\delta CuO_4$, which is hole-doped, the sign of R_H changes from positive to negative at $\delta \approx 0.3$ [2, 3]. Similarly, for the electron-doped material $Nd_{2-\delta}Ce_\delta CuO_4$, R_H changes sign at $\delta \approx 0.16$, but now from negative to positive, displaying a clear symmetry with the hole-doped example [2].

The dependence of the magnitude of R_H on doping level is also of interest for the high-T_c cuprates. In $La_{2-\delta}Sr_\delta CuO_4$, at very low doping levels, $R_H \propto 1/\delta$, for δ up to about 0.1 [2, 3] or 0.05 [4], but decreasing more rapidly than $1/\delta$ for higher values of the doping. The electron-doped $Nd_{2-\delta}Ce_\delta CuO_4$ shows a similar dependence on δ too, albeit with opposite sign.

Many different theories have been proposed to explain the curious features of the Hall effect in the high-T_c materials, but all fall into one of two groups: those employing a Fermi liquid approach, where changes in the topology of a Fermi surface are studied, and those in which models in the strong-coupling limit are examined; we shall consider the former category first. Kim *et al* [5] considered the d–p model for the CuO_2 planes using the slave-boson method, where in order to obtain a value of R_H that was positive for low doping levels, the bare tight-binding band structure had to be modified to include direct in-plane oxygen–oxygen overlap (hopping) terms

0953-8984/93/030023+06$07.50

(else $R_{\rm H}$ would have been negative). However, for a strictly two-dimensional system they found non-singular behaviour for $R_{\rm H}$, calculated using the Boltzmann equation, in the low-doping limit, which they were only able to rectify by changing to a three-dimensional band structure instead. Another approach has been given by Melo *et al* [6], who investigated the Hall effect in the t–J model using a slave-boson approach combined with large-N expansion. They showed that by carefully treating the short-range antiferromagnetic correlations present, it was possible to obtain an $R_{\rm H}$, again determined using the Boltzmann equation (following Kohno and Yamada [23]), that changed sign, from positive to negative, with increasing doping. However, the method was unsuitable for examining the low-doping (Mott–Hubbard) regime, where $R_{\rm H}$ diverges and no mention of the temperature dependence was made. A third point of view has been proposed by Voruganti *et al* [7] in studying the two-dimensional Hubbard model. For non-zero doping and using incommensurate planar spin-density-wave (spiral) saddle points they found that, due to the associated change in the Fermi surface, $R_{\rm H}$ changed sign for $U/t = 2$–10 at dopings $\delta = 0.02$–0.5. They also found $R_{\rm H}^{-1}$ to be a monotonically increasing function of temperature, being practically a constant for low temperatures, but rapidly increasing when the temperature implied a sign change of $R_{\rm H}$, which corresponded to a transition from a spiral spin state, with $R_{\rm H} > 0$, to the antiferromagnetic state, where $R_{\rm H} < 0$. However, their model could not be used to account for the linear resistivity characteristic of the high-$T_{\rm c}$ systems.

The other approach is based, typically, on a t–J model in the continuum limit using the slave representation, consisting of fermionic ($\overline{y}^{\alpha}, y^{\alpha}; \alpha = \uparrow, \downarrow$) and bosonic ($\overline{y}^{0}, y^{0}$) slaves, interacting with each other through an internal gauge field (a) [8–10]. (The appearance of the gauge field is a reflection of the local phase symmetry of the t–J Hamiltonian.) The response of the electrons, the physical particles of the system, may then be studied by combining the slaves back into the electrons first and then perturbing the combined particle with the relevant external probes, but this method is difficult [11]. In an alternative, but physically equivalent and mathematically tractable method, the slaves are probed directly and then the no-double-occupancy constraint enforces a relationship between the electronic and unphysical slave responses. This approach was pioneered by Ioffe and Larkin [8] and has been successful in predicting a resistivity that displays a linear temperature dependence, as well as applying to several other problems [13–15], including the Hall effect, where it predicts that $R_{\rm H} \sim \delta^{-1}$, $\delta \ll 1$ [13] and the correct qualitative temperature dependence of $R_{\rm H}$ may be obtained as a signature of the breaking of parity [15]. However, we have been unable to find any first-principles derivation of the Hall current using the gauge approach, and so in this work we present such a calculation.

The (imaginary time) Lagrange function at the supersymmetric point $J = 2t$ given in [10] reads

$$\mathcal{L} = \sum_{a=0,\uparrow,\downarrow} \overline{y}^{a} \left(\partial_{\tau} - \frac{J}{2}(\nabla + {\rm i}a)^2 \right) y^{a} \qquad (1)$$

where the fermionic and bosonic slaves are subject to $\sum_{a=0,\uparrow,\downarrow} \overline{y}^{a} y^{a} = 1$. Observe that a has no dynamics and can be expressed in terms of the slave currents through the field equation $\partial\mathcal{L}/\partial a = 0$ and is given dynamics by integrating out the fermionic and bosonic slaves. Note that we have adopted the potential gauge $a_0 = 0$ [16]. This is a non-relativistic non-linear σ-model with one bosonic and two fermionic directions. The action ($\int \mathcal{L}$) gives equal bare masses for the slaves: $m_{\rm F} = m_{\rm B} = J/b^2$, b being the lattice spacing [10]. To investigate the Hall effect we couple an external vector potential (A) into the system and then find the effective action through to third order [17] in A, since the Hall current is proportional to the product of the external electric and magnetic fields [18].

To do this we first integrate out the holons and spinons to give the action of the internal gauge field, which may be expressed as a power series of this field, with coefficients that are each the sum of a holon and a spinon term. The external field is then coupled to the holon and the effective action of A is calculated by integrating out the internal gauge field. A saddle point procedure followed by solving the induced field in a series development of the external field, which should be accurate enough to describe the response of the system to weak external fields, gives us the effective action and is found to be (using the 'super-condensed notation' where a suffix denotes a discrete element and a spacetime coordinate, e.g. $A_a \leftrightarrow A_\alpha(x,\tau)$ and the repeated suffices are summed and/or integrated over)

$$S^{\mathrm{eff}}[A] = \frac{1}{2}\left(\frac{e}{c}\right)^2 A_a \Pi^{\mathrm{eff}}_{ab} A_b + \frac{1}{3}\left(\frac{e}{c}\right)^3 A_a A_b A_c \Gamma^{\mathrm{eff}}_{abc} \tag{2}$$

the charge e being the charge on the holon (see below). $\Pi^{\mathrm{eff}}_{\mu\nu}$ is given by the Ioffe–Larkin result [8], $\Pi^{\mathrm{eff}} = \Pi^{\mathrm{B}}\Pi^{\mathrm{F}}/(\Pi^{\mathrm{B}}+\Pi^{\mathrm{F}})$ and $\Gamma^{\mathrm{eff}}_{\alpha\beta\gamma}$ is the three-field vertex; with the notation $\zeta_{\mathrm{B/F}} = +1/-1$, it is

$$\Gamma^{\mathrm{eff}}_{abc} = \sum_{s=\mathrm{B,F}} \zeta_s \Pi^{s}_{ad}\mathcal{D}_{dk}\Pi^{s}_{be}\mathcal{D}_{el}\Pi^{s}_{cf}\mathcal{D}_{fm}\Gamma^{s}_{klm} \tag{3}$$

where Γ^s is the three-field vertex for the slave of species s and may be obtained from third-order perturbation theory, giving

$$\begin{aligned}\Gamma^{s}_{\alpha\beta\gamma}(x_1,x_2,x_3) = \frac{1}{2}(\langle J^s_\alpha(x_1)J^s_\beta(x_2)J^s_\gamma(x_3)\rangle \\ -\frac{3}{m_s}\delta_{\alpha\beta}\delta(x_1-x_2)\langle\rho^s(x_1)J^s_\gamma(x_3)\rangle)\end{aligned} \tag{4}$$

(which may be easily written in a more symmetric form) J^s and ρ^s being the s-slave current and density respectively. $\mathcal{D} = (\Pi_{\mathrm{B}}+\Pi_{\mathrm{F}})^{-1}$ is the internal gauge-field Green's function and Π^s is the s-slave polarization [10]. Note that if the spinons had been charged instead, then we would interchange B and F in Π^{eff} and Γ^{eff}, which leaves Π^{eff} unaltered, but introduces a sign change in Γ^{eff}. This is correct, since if the physical electron's charge is $-e$ ($e>0$), then the holon charge will be e, while the charge on the spinon would be $-e$. Therefore, in charging the spinons instead, we change the sign of e, so e^3 becomes $-e^3$, cancelling the sign change in Γ^{eff}, and hence the action is unchanged [19]. If the y^a were the physical field of the problem, the screening factor $[\Pi\mathcal{D}]^3$ would not appear. Such factors may be interpreted as the reaction of the system to A.

The physical current (J) is the derivative of the action with respect to A/c and is given by

$$J_a = \frac{e^2}{c}\Pi^{\mathrm{eff}}_{ab}A_b + \frac{e^3}{c^2}\Gamma^{\mathrm{eff}}_{abc}A_bA_c. \tag{5}$$

To calculate the Hall current we follow [21] and take $\boldsymbol{A}$ to be the sum of two fields, $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$, where the former corresponds to an electric field, $\mathrm{i}\omega \boldsymbol{A}^{(1)}/c = \boldsymbol{E}$ (real time) and the latter to a magnetic field, $\mathrm{i}\boldsymbol{k} \times \boldsymbol{A}^{(2)} = \boldsymbol{B}$. For $\boldsymbol{E} = (0, E, 0)$ and $\boldsymbol{B} = (0, 0, B)$, the current in the 1-direction linear in both these fields is the Hall current (J_1^{H}) and is given by $J_1^{\mathrm{H}} = \sigma_{\mathrm{H}}[\boldsymbol{E} \times \boldsymbol{B}]_1$, σ_{H} being the Hall conductivity, in the limit $\omega, k \to 0$, such that $k/\omega \to 0$.

To determine σ_{H} we must calculate the three-field vertices for the slaves. The temperature (T) of the system we take to be below the spinon's Fermi temperature, $T_{\mathrm{F}} = \pi n_{\mathrm{F}}/m_{\mathrm{F}}$, but above the Bose temperature of the holons, $T_{\mathrm{B}} = n_{\mathrm{B}}/m_{\mathrm{B}}$, where $n_{\mathrm{B}} b^2 = \delta$ and $n_{\mathrm{F}} b^2 = 1 - \delta$, δ being the hole filling fraction. Then, for only the simplest diagrams, but introducing a transport lifetime (τ_s) for the slaves, arising from dressing the slaves with the internal gauge field, and analytically continuing to real frequencies, in the small-ω and small-$\boldsymbol{k}$ limit, for the spinons, $\Gamma^{\mathrm{F}}_{\alpha\beta\gamma} = [\delta_{\alpha\beta} k_\gamma - \delta_{\beta\gamma} k_\alpha]\omega n_{\mathrm{F}}(\tau_{\mathrm{F}}/m_{\mathrm{F}})^2/2$, which we take for the holons as well. This calculation is based upon those in [22, 23], wherein it is shown that for an isotropic system, the tensor structure of the Γ is unaffected by including all possible graphs in its calculation (for a Fermi liquid). It follows that we are led to consider the term $[\Pi^s D]_{\beta\nu}(\mathbf{0}, -\omega)$, D being the real-time gauge-field propagator, which arises because the E-field is spatially uniform. We calculate this as $[\Pi^s D]_{\beta\nu}(-\kappa\boldsymbol{k}, -\omega)$ for $\kappa \to 0$ and thus obtain

$$\sigma_{\mathrm{H}} = \frac{e^3}{c} \lim_{\omega\to 0} \lim_{k\to 0} \sum_s \zeta_s [\Pi^s D]_{\mathrm{T}}(-\boldsymbol{k}, 0)[\Pi^s D]_{\mathrm{L}}(\boldsymbol{k}, \omega)[\Pi^s D]_{\mathrm{T}}(\mathbf{0}, -\omega) n_s \left(\frac{\tau_s}{m_s}\right)^2 \quad (6)$$

where L and T stand for longitudinal and transverse parts, respectively, and in the limits shown $[\Pi^s D]_{\mathrm{T}}(\mathbf{0}^{\pm}, 0) = \chi_s/(\chi_{\mathrm{B}} + \chi_{\mathrm{F}})$, χ_s being the diamagnetic susceptibility of the s-slave and $[\Pi^s D]_{\mathrm{L/T}}(\mathbf{0}, 0^{\pm}) = \sigma_s/(\sigma_{\mathrm{B}} + \sigma_{\mathrm{F}})$, where σ_s is the conductivity of the s-slave. These last results follow from $\Pi^s_{\mathrm{T}}(\mathbf{0}, \omega) \approx -\mathrm{i}\sigma_s\omega$ and $\epsilon_s = \Pi^s_{\mathrm{L}}/\omega^2$, with the slave permittivity related to the conductivity by $\epsilon_s(\omega) \propto \sigma_s/\mathrm{i}\omega$ [20]. Note that no initial photon is present here, which implies strict proportionality, but in three dimensions the analogous relations ensure that the permittivity tends to the correct high-frequency result, satisfying the plasma sum rule. Thus,

$$\sigma_{\mathrm{H}} = \frac{e^3}{c}\left[n_{\mathrm{B}}\left(\frac{\tau_{\mathrm{B}}}{m_{\mathrm{B}}}\right)^2 \left[\frac{\sigma_{\mathrm{F}}}{\sigma_{\mathrm{B}} + \sigma_{\mathrm{F}}}\right]^2 \frac{\chi_{\mathrm{F}}}{\chi_{\mathrm{B}} + \chi_{\mathrm{F}}} - n_{\mathrm{F}}\left(\frac{\tau_{\mathrm{F}}}{m_{\mathrm{F}}}\right)^2 \left[\frac{\sigma_{\mathrm{B}}}{\sigma_{\mathrm{B}} + \sigma_{\mathrm{F}}}\right]^2 \frac{\chi_{\mathrm{B}}}{\chi_{\mathrm{B}} + \chi_{\mathrm{F}}}\right]. \quad (7)$$

In the temperature range used, $\chi_{\mathrm{B}} = n_{\mathrm{B}}/12 m_{\mathrm{B}}^2 T$, while $\chi_{\mathrm{F}} = 1/12\pi m_{\mathrm{F}}$. For the conductivities, if we take the classical result, $\sigma_s = e^2 n_s \tau_s / m_s$, then we see that $\sigma_{\mathrm{H}} = 0$ *for* $\delta = 0$ *and* $\delta = 1$. *Since both the undoped* ($\delta = 0$) *and fully doped* ($\delta = 1$) cases are insulating, these results are correct. However, the fully doped Hall conductivity is not strictly obtainable from the expression quoted as it would require $T < T_{\mathrm{B}}$, which is out of the temperature range considered.

The Hall coefficient is given by $R_{\mathrm{H}} = \sigma_{\mathrm{H}}/(\sigma^{\mathrm{eff}})^2$ and reads

$$R_{\mathrm{H}} = \frac{1}{ec} \frac{R^{\mathrm{B}}_{\mathrm{H}}\chi_{\mathrm{F}} + R^{\mathrm{F}}_{\mathrm{H}}\chi_{\mathrm{B}}}{\chi_{\mathrm{B}} + \chi_{\mathrm{F}}} \quad (8)$$

where R^s_{H} is the Hall coefficient of the s-slave and for our simple calculation of the three-field vertices is given by $R^s_{\mathrm{H}} = \zeta_s / n_s$.

Equation (8) was mentioned in [13–15] and not derived therein. Where some explanation has been forthcoming, reference to [8] has been made [15], but this can only yield results for responses of the system that are linear in the external fields, while the Hall effect is second order.

In the low-doping limit, we find that

$$R_{\mathrm{H}} \approx \frac{1}{n_{\mathrm{B}}ec} \qquad \delta \ll 1 \tag{9}$$

which is independent of temperature. In fact, for $\delta \ll 1$, where $\chi_{\mathrm{F}} \gg \chi_{\mathrm{B}}$, the linear resistivity regime, $R_{\mathrm{H}} \approx R_{\mathrm{H}}^{\mathrm{B}}/ec$. Therefore, the temperature dependence of R_{H} must come from $R_{\mathrm{H}}^{\mathrm{B}}$, at very low doping levels. Considering higher-order diagrams, including an internal gauge field with an odd-parity element, has been shown to give to a leading-order temperature dependence [15], as a signal of the spontaneous generation of magnetic field leading to parity breaking. As mentioned at the beginning, the temperature dependence can be a strong function of doping, as seen in $La_{2-\delta}Sr_{\delta}CuO_4$, where for very low doping levels practically no temperature dependence is observed. For higher doping levels, consistent with the temperature, we find that R_{H} increases with temperature, which is contrary to experiment.

Another feature of the Hall coefficient of the hole-doped high-T_c materials that we may discuss is the change of sign of R_{H} with increased doping. At the supersymmetric point, R_{H} changes sign, from positive to negative, between $T/J = 0.25$, at $\delta = 0.245$ and $T/J = 1.57$, at $\delta = 0.500$, consistent with the temperature range where our calculation is valid, the doping being $2\pi\delta_c = [(4\pi J/T + 1)^{1/2} - 1]T/J$. Thus we see that despite changing sign over quite a wide range of temperatures, δ_c does not vary as much.

Finally, we may consider the curious temperature regime $T \gg T_{\mathrm{F}}, T_{\mathrm{B}}$. Now $\chi_s = n_s/12m_s^2T$ for both holons and spinons. At the supersymmetric point [10], $R_{\mathrm{H}} \propto (1 - 2\delta)/\delta(1 - \delta)$, which vanishes at $\delta = 1/2$, i.e. quarter filling, when the numbers of holons and spinons are equal. Under the assumption of equal masses and such high temperatures—so both slave species obey Boltzmann statistics—the system consists of two fluids that have opposite slave charges, but are otherwise identical. Thus the fluids have equal and opposite Hall currents, the sum of which, and hence the Hall coefficient of which, is zero.

In conclusion, we have determined the effective action of the t–J model coupled to an external electromagnetic field, in the slave representation, to third order in the field. We find that the Ioffe–Larkin result for the second-order action is unchanged and that the third-order term is, correctly, independent of which slave species is charged. From the action we have derived an expression for the Hall conductivity, which we explicitly calculate using the simplest set of graphs. This leads to a Hall coefficient that, for very low doping, is positive and inversely proportional to the doping and is independent of temperature. For higher levels of doping the Hall coefficient increases with temperature, which disagrees with the experimental results. However, we can obtain a change of sign with doping at temperatures within the temperature range considered.

Throughout we have assumed the slave masses to be given by the bare masses, but we expect that a higher-order calculation would enhance the boson mass thereby depressing the Bose temperature and could give the Hall number the observed temperature dependence.

We should like to thank D M Edwards, A C Hewson, D K K Lee and D M Newns for helpful discussions during the course of this work. SMM would like to thank the Science and Engineering Research Council (SERC), UK, for financial support during the course of this work.

Note added in proof. After submission we became aware of two further studies of the Hall effect using the gauge field approach: a very recent paper using essentially the method described above [24] and a paper by Lee and Nagaosa [25], in which only the diagram for the three-current correlator contribution to σ_H was shown.

References

[1] Chien T R, Wang Z Z and Ong N P 1991 *Phys. Rev. Lett.* **67** 2088
[2] Burns G 1992 *High-Temperature Superconductivity–An Introduction* (London: Academic)
[3] Suzuki M 1989 *Phys. Rev.* B **39** 2312
[4] Takagi H, Ido T, Ishibashi S, Uota M, Uchida S and Tokura Y 1989 *Phys. Rev.* B **40** 2254
[5] Kim J H, Levin K and Auerbach A 1989 *Phys. Rev.* B **39** 11633
[6] Sá de Melo C A R, Wang Z and Doniach S 1992 *Phys. Rev. Lett.* **68** 2078
[7] Voruganti P, Golubenstev A and John S 1992 *Phys. Rev* B **45** 13945
[8] Ioffe L B and Larkin A I 1989 *Phys. Rev* B **39** 8988
[9] Nagaosa N and Lee P A 1990 *Phys. Rev. Lett.* **64** 2450
[10] Chen Y and Förster D 1992 *Phys. Rev.* B **45** 938
[11] Chen Y, Förster D and Larkin A I 1992 *Phys. Rev.* B **46** 5370
[12] Ioffe L B and Wiegmann P B 1990 *Phys. Rev. Lett.* **65** 653
[13] Nagaosa N and Lee P A 1991 *Phys Rev.* B **43** 1233
[14] Ioffe L B and Kotliar G 1990 *Phys. Rev.* B **42** 653
[15] Ioffe L B, Kalmeyer V and Wiegmann P B 1991 *Phys. Rev.* B **43** 1219
[16] Abrikosov A A, Gorkov L P and Dzyaloshinskii I E 1975 *Methods of Quantum Field Theory in Statistical Physics* (London: Dover)
[17] The effective action as a functional of the external field (sources) can be systematically constructed, see
Schwinger J 1951 *Proc. Natl. Acad. Sci. USA* **37** 455
The method used there is applied to our problem.
[18] In a charge-conjugation symmetric system such as the relativistic spinor electrodynamics, the effective action can only be expanded in an even functional power series in A_ν which is encapsulated in a theorem due to Furry
Furry W H 1937 *Phys. Rev.* **51** 125
The leading non-linear correction to the Maxwell theory in the long-wavelength limit is quartic in the external field and proportional to e^4; see
Heisenberg W and Euler H 1936 *Z. Phys.* **98** 174
Schwinger J 1951 *Phys. Rev.* **82** 664
[19] We should like to emphasize that the B $\leftrightarrow$ F anti-symmetry expressed in (4) is a generic feature of the slave/gauge approach to the t–J model, independent of δ, T, J etc. It would be somewhat odd to have a situation where one of the terms vanishes identically. A charge-conjugation-symmetric theory must contain expansion even powers of e in its effective action.
[20] The relationship with the polarization is well known; for a thorough treatment see
Martin P C 1967 *Phys. Rev.* **161** 143
[21] Altshuler B I, Khmel'nitzkiĭ D, Larkin A I and Lee P A 1980 *Phys. Rev.* B **22** 5142
[22] Fukuyama H, Ebisawa H and Wada Y 1969 *Prog. Theor. Phys.* **42** 494
[23] Kohno H and Yamada K 1988 *Prog. Theor. Phys.* **80** 623
[24] Schofield A J and Wheately J M 1992 *Phys. Rev.* B submitted
[25] Lee P A and Nagaosa N 1992 *Phys. Rev.* B **46** 5621